Analysis of microchannel heat exchangers using FEM

G.A. Quadir, Anvar Mydin and K.N. Seetharamu
School of Mechanical Engineering, University of Science, Perak, Malaysia

Received April 2000
Revised September 2000
Accepted September 2000

Keywords *Finite element method, Heat sinks, Performance measurement, Flux, Heat exchanger*

Abstract *A finite element method is applied to evaluate the performance of microchannel heat exchangers that are used in electronic packaging. The present approach is validated against the CFD data available in the literature. A comparison of the predicted results with other available results obtained from different concepts shows that the present method is able to predict the surface temperature, the fluid temperature and thus the total thermal resistance of the microchannel heat sink satisfactorily. The present methodology has an added advantage in that non-uniform surface heat flux distribution over the package base can also be analysed easily. The method used in the present analysis is an alternative to massive CFD calculations.*

Nomenclature

C_p = Specific heat at constant pressure, J/kgK
D = Hydraulic diameter of the channel, m
f = Friction factor
G = Total volume flow rate of the fluid, m^3/s
H_c = Microchannel height, m
h_c = Heat transfer coefficient, W/m^2K
k = Thermal conductivity, W/mK
L = Total channel length, m
$\dot{m}$ = Mass flow rate of the coolant, kg/s
N_1, N_2 = Shape functions
NE = Number of elements
Nu = Nusselt number
n = Number of channels
POW = Pumping power, W
ΔP = Pressure drop along channel length, Pa
Q = Total heat flux, W/m^2
q = Heat flux, elementwise, W/m^2
Pr = Prandtl number
Re = Reynolds number
R_{Total} = Total Thermal Resistance, °C/W
T = Temperature, °C
t = Fin half thickness, m
V = Fluid velocity, m/s
W = Total width of the heat sink, m
w_c = Channel width, m
x = Flow/heat transfer direction along channel length, m
z = Heat transfer direction along channel height, m
ρ = Density, kg/m^3
α = Aspect ratio = H_c/w_c
β = Fin width to channel width ratio
η = Fin efficiency
μ = Dynamic viscosity of the fluid, Ns/m^2
ε = Porosity

Introduction

Miniaturisation of electronic systems is progressing at a rapid rate. The reliability of an electronic system depends to a major extent on the temperature level in and around the components. Thus the thermal management becomes an important issue as the size of the components is reduced with the attendant increase in the heat dissipation rate. Thermal management of nanometer scale electronic devices becomes an even more important issue (Goldhaber-Gordon *et al.*, 1997). For a low heat flux system, heat is conducted from the sources to the packaged surface where it is removed by natural or forced flow of ambient air together with radiation. However, in high heat flux systems, forced and free convection of a single or two phase fluid, immersion cooling, jet impingement and cryogenic methods have been successfully used (Daane and Franzon, 1993) as cooling techniques in the electronics industry.

Microchannels are used as heat sinks for the purpose of cooling electronic chips as they have high heat transfer coefficients. Tuckerman and Pease (1981) were the first to apply this idea to the cooling of VLSI chips. They fabricated a microchannel by etching them directly on the back of silicon IC chips and a cover plate at the fin tips was used to confine the coolant in the channel. Harpole and Eninger (1991) proposed to have between ten and 30 manifold channels per centimeter of chip length in order to reduce the maximum temperature of heat sources at constant flow rate and pumping power. Wang and Peng (1994) investigated experimentally single phase forced convection of water or methanol flowing through microchannel with rectangular cross section. Mizunuma *et al.* (1997) conducted experiments to investigate the forced convective heat transfer from a micro-finned surface to a fluorocarbon liquid FX3250 and carried out numerical simulation of flow and heat transfer as well.

Salman (1989) studied convective transfer in a microchannel analytically under the assumptions of constant heat flux to the fluid with hydrodynamically and thermally fully developed flow. Recently Zhimin and Fah (1997) analyzed the flow and heat transfer in microchannel heat sinks through a thermal resistance model assuming uniform distribution of heat load on the base of the heat sink. Their model considered the sum of thermal resistances due to the bulk temperature rise, convection, constriction and conduction expressed as:

$$R_{Total} = R_{Bulk} + R_{Conv} + R_{Constr} + R_{Cond} \tag{1}$$

Considering the coolant and heat sink material properties, the flow and heat transfer status and the heat sink's geometry, the above expression was summarized as:

$$\begin{aligned} R_{Total} = {} & \frac{L}{C_{pf}\,\mu_f}\frac{2}{Re}\frac{1+\beta}{1+\alpha} + \frac{1}{Nu\,k_f}\frac{1+\beta}{1+2\,\alpha\,\eta}\frac{2\alpha}{1+\alpha}\,w_c \\ & + \frac{1+\beta}{\pi\,k_s}\ln\left[\frac{1}{Sin\,\frac{\pi\,\beta}{2\,(1+\beta)}}\right]w_c + \frac{1}{k_s} \end{aligned} \tag{2}$$

They studied the optimum thermal design of the heat sink over wide flow and heat transfer regimes using their model under three flow constraints, constant volume flow rate, constant pressure drop and constant pumping power. Knight *et al.* (1991) reported a design method for microchannel heat sinks, which gave 35 per cent lower thermal resistance as compared to that obtained by Tuckerman and Pease (1981). The microchannel was modeled as a porous medium by Koh and Colony (1986) where Darcy's law was used to describe the flow. Later, the Darcy equation, modified by Vafai and Tien (1981) and two-equation model for heat transfer was used by Tien and Kuo (1987) in their proposed model. They reported an analytical solution for the velocity distribution and a numerical solution for the temperature distribution. Recently, Kim and Kim (1999) reported analytical solutions for both velocity and temperature profiles in microchannel heat sinks by modeling the microchannel heat sink as a fluid-saturated porous medium. They gave an expression for the total thermal resistance which was derived from the analytical solutions and the geometry of the microchannel heat sink for which the thermal resistance of the heat sink is minimal.

Knight *et al.* (1992), while studying the performance of an air-cooled heat sink found much higher thermal resistance at a given pressure drop. Weisberg *et al.* (1992) and Yeung *et al.* (1995) used numerical methods to solve the conjugate heat transfer problem in the fluid and solid of a microchannel. Commercial CFD software was used by Copeland *et al.* (1996) to solve the heat transfer and the flow problem in a manifold microchannel heat sink. Leng (1997) investigated the heat transfer and flow problem in a microchannel using FLUENT software. He studied the effect of various geometric characteristics of the flow channel, fins and substrate on the thermal resistance as a function of coolant flow rate.

In a microchannel heat exchanger, the effects of longitudinal heat conduction as well as conduction along the fin height have to be considered in the analysis. Recent research publications (Ranganayakulu and Seetharamu, 1999a, 1999b) by one of the authors considered the combined effects of longitudinal heat conduction, flow non-uniformity and temperature non-uniformity in cross flow plate-fin heat exchangers and compact tube-fin heat exchangers using the finite element method.

Method of analysis

Modern microfabrication technology is capable of fabricating microchannels that are used as heat sinks for the cooling of electronic chips. The general structure of a microchannel is shown in Figure 1.

A uniform/non-uniform heat flux from the chip may be imposed at the lower surface of the base, whereas, the upper surface contains a series of rectangular channels carrying coolant. The channels are separated by fins. A cover plate is bonded to the top of the channel and fin array to confine the coolant.

The first step in the numerical solution of any differential equation is to discretise the domain over which the solution is sought. The microchannel is divided into a number of elements along the longitudinal direction, i.e. length direction. A typical element is shown in Figure 2 along with the nodes and the heat transfer mode. The walls are discretised by four noded bilinear elements. Conduction heat transfer is considered in two directions in both the vertical walls. The wall 1,2,8,7 receives heat from the source and loses heat to the coolant fluid by convection directly from the base and indirectly through side

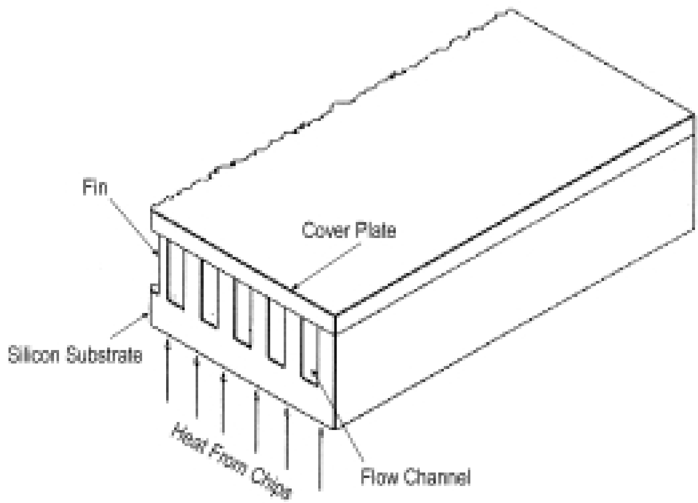

Figure 1.
General structure of a microchannel heat sink

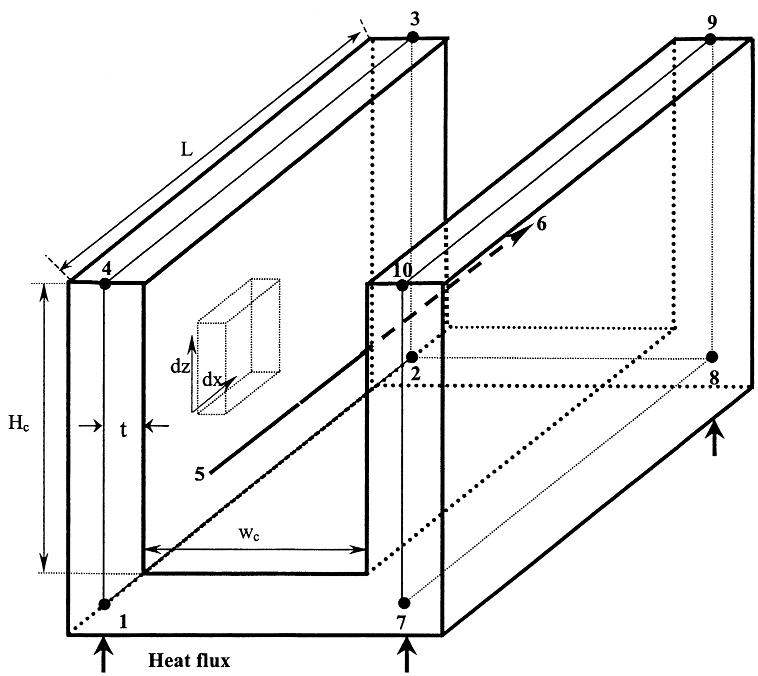

Figure 2.
Details of an element

walls. As a first step, a uniform heat load is distributed at the four corners of the base of the substrate equally. If the source is not uniform, the nodes will receive unequal heat loads. An assembly of elements in the length direction will complete the full channel. The method of discretisation is very general and is capable of extending either longitudinally or transversely, or in both directions.

Governing equation and finite element formulation

Considering the control volume in the left wall of Figure 2 and the energy balance including the conduction and convection heat transfer yields the governing equation:

$$k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial z^2} - \frac{h_c}{t}(T - T_f)_{Left} = 0 \tag{3}$$

Similarly the governing equation for the right hand side wall, can be written as:

$$k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial z^2} - \frac{h_c}{t}(T - T_f)_{Right} = 0 \tag{4}$$

The heat transferred from the two walls and the bottom surface to the fluid element results in the governing equation for the fluid as:

$$\begin{aligned} &\dot{m}\,C_p\frac{dT_f}{dx} - h_c H_c (T - T_f)_{Left} - h_c H_c (T - T_f)_{Right} \\ &- h_c w_c (T - T_f)_{Bottom} = 0 \end{aligned} \tag{5}$$

It may be noted that no separate differential equation has been written for the energy balance on the base (including conduction and convection) to determine T_1, T_2, T_7 and T_8 as these temperatures can be calculated using the equations (3) to (5). The finite element formulation of these equations was carried out by the Galerkin method. In the weighted residual integral over the domain, it was assumed that the temperature driving potential for convective heat transfer $(T - T_f)$ is the difference between the average temperatures of the wall and the fluid with which the wall is interacting. The following equivalents are, therefore introduced at this stage.

$$(T - T_f)_{Left} = \frac{1}{4}\sum_{i=1}^{i=4} T_i - \frac{1}{2}\sum_{j=5}^{j=6} T_j \tag{6}$$

$$(T - T_f)_{Right} = \frac{1}{4}\sum_{i=7}^{i=10} T_i - \frac{1}{2}\sum_{j=5}^{j=6} T_j \tag{7}$$

$$(T - T_f)_{Bottom} = \frac{1}{4}(T_1 + T_2 + T_7 + T_8) - \frac{1}{2}\sum_{j=5}^{j=6} T_j \tag{8}$$

The wall temperature is assumed to vary as

$$T = \sum_{i=1}^{i=4} N_i T_i$$

where the shape functions N_i for a general quadrilateral element (the rectangular element being a particular case) are given by the expression:

$$N_i = (1 + \xi\xi_i)(1 + \eta\eta_i)$$

where ξ_{i}, η_{i} are the natural co-ordinates of nodes of the rectangular element and ξ, η are the natural coordinates of any point in the rectangular element. The fluid temperature is assumed to vary as:

$$T_f = \sum_{j=5}^{j=6} N_j T_j$$

where the shape functions N_i and N_j for the linear element are given by

$$Ni = 1 - x/L, Nj = x/L.$$

Applying the Galerkin weighted residual integrals to equations (3), (4), and (5) a complete set of algebraic equations for the element is written as:

$$\begin{aligned}&(2e_x + 2e_z + c)T_1 + (-2e_x + e_z + c)T_2 + (-e_x - e_z + c)T_3 \\ &+ (e_x - 2e_z + c)T_4 - 2c(T_5 + T_6) = 0\end{aligned} \tag{9}$$

$$\begin{aligned}&(-2e_x + e_z + c)T_1 + (2e_x + 2e_z + c)T_2 + (e_x - 2e_z + c)T_3 \\ &+ (-e_x - e_z + c)T_4 - 2c(T_5 + T_6) = 0\end{aligned} \tag{10}$$

$$\begin{aligned}&(-e_x - e_z + c)T_1 + (e_x - 2e_z + c)T_2 + (2e_x + 2e_z + c)T_3 \\ &+ (-2e_x + e_z + c)T_4 - 2c(T_5 + T_6) = 0\end{aligned} \tag{11}$$

$$\begin{aligned}&(e_x - 2e_z + c)T_1 + (-e_x - e_z + c)T_2 + (-2e_x + e_z + c)T_3 \\ &+ (2e_x + 2e_z + c)T_4 - 2c(T_5 + T_6) = 0\end{aligned} \tag{12}$$

$$\begin{aligned}&(U + W)(T_1 + T_2) + U(T_3 + T_4) - 2(U + V + W)(T_5 + T_6) \\ &- f_m(T_5 - T_6) + (V + W)(T_7 + T_8) + V(T_9 + T_{10}) = 0\end{aligned} \tag{13}$$

$$\begin{aligned}&(U + W)(T_1 + T_2) + U(T_3 + T_4) - 2(U + V + W)(T_5 + T_6) \\ &- f_m(T_5 - T_6) + (V + W)(T_7 + T_8) + V(T_9 + T_{10}) = 0\end{aligned} \tag{14}$$

$$\begin{aligned}&- 2c(T_5 + T_6) + (2e_x + 2e_z + c)T_7 + (-2e_x + e_z + c)T_8 \\ &+ (-e_x - e_z + c)T_9 + (e_x - 2e_z + c)T_{10} = 0\end{aligned} \tag{15}$$

$$\begin{aligned}&- 2c(T_5 + T_6) + (-2e_x + e_z + c)T_7 + (2e_x + 2e_z + c)T_8 \\ &+ (e_x - 2e_z + c)T_9 + (-e_x - e_z + c)T_{10} = 0\end{aligned} \tag{16}$$

$$\begin{aligned}&- 2c(T_5 + T_6) + (-e_x - e_z + c)T_7 + (e_x - 2e_z + c)T_8 + (2e_x \\ &+ 2e_z + c)T_9 + (-2e_x + e_z + c)T_{10} = 0\end{aligned} \tag{17}$$

$$
\begin{aligned}
&-2c(T_5+T_6)+(e_x-2e_z+c)T_7+(-e_x-e_z+c)T_8 \\
&+(-2e_x+e_z+c)T_9+(2e_x+2e_z+c)T_{10}=0
\end{aligned}
\tag{18}
$$

where,

$c = h_c L H_c/16$
$e_x = tkH_c/6L$
$e_z = tkL/6H_c$
$f_m = \dot{m}\, C_p/2$
$U = -h_c L H_c/8$
$V = -h_c H_c L/8$
$W = -h_c w_c L/8$

The present element matrix has ten nodes and therefore it will be a 10×10 matrix. The element matrices are stored in the proper order to obtain the global stiffness matrix as explained in Segerlind (1984) and Lewis *et al.* (1996). Thus the global stiffness matrix for eight elements (as shown in Figure 3) will have 45 nodal equations and therefore will be a 45×45 matrix. This is solved by Gauss elimination to get the wall and coolant temperatures.

Results and discussion

Validation

In order to use a methodology developed for a particular problem with more confidence, generally the output has to be validated against some established or known results, preferably with experimental results obtained under the same conditions or with results obtained by other numerical or analytical methods.

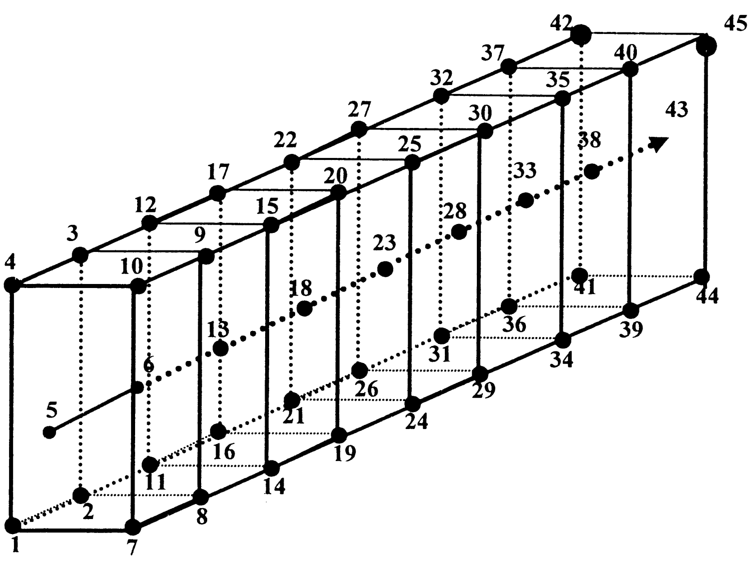

Figure 3.
Node points for eight elements

In the present analysis, the comparison will be made first with the recent CFD results for the same dimension microchannel heat exchangers analysed under the same conditions and reported by Leng (1997).

Before the results are validated an attempt is made to establish the minimum number of elements required to produce element independent results. For this the computation was carried out for the following channel dimensions and fluid properties with different numbers of elements:

- Channel height = 200 μm.
- Channel width = 56 μm.
- Fin half thickness = 12 μm.
- Channel length = 0.5 cm.
- Coolant = water.
- Fluid velocity = 1.973 m/s (Leng, 1997).
- Overall hc = 54,290 W/m^{2}K (Leng, 1997).
- Coolant inlet temperature = 10 °C.
- Channel thermal conductivity (silicon) = 125 W/mK.
- Coolant thermal conductivity (k_f) = 0.609 W/mK
- Coolant absolute viscosity(μ_f) = 9.8×10^{-4} N s/m^2.
- Coolant specific heat capacity (C_{pf}) = 4180 J/kgK.
- Coolant density (ρ_f) = 1,000 kg/m^3.

Table I shows the surface maximum temperature as a function of the number of elements. It can be observed that when the number of elements is eight and beyond, $T_{sur,max}$ is found to be independent of the number of elements used during computation. It is interesting to note that the solution with four elements (20.043 °C) is only differing from the eight element solution by 0.02 per cent. However, all calculations that follow are based on eight elements for the microchannel.

It may be mentioned here that the total base heat load was divided equally between the elements considered in the analysis. The four bottom surface nodes of each element thus receive one fourth of the element heat load. The surface maximum temperature and the fluid outlet temperature for the above channel dimensions are found to be 20.006 °C and 15.16 °C respectively. The percentage deviations for the present prediction of surface temperature and the fluid outlet temperature with respect to the CFD results (Leng, 1997) are found to be less than 2.5 per cent. Thus the present methodology predicts the performance successfully.

Table I. Surface maximum temperature variation with different number of elements

Number of elements	Maximum surface temperature(°C), $T_{sur,max}$
1	17.6
2	20.14
4	20.043
8	20.006
16	20.006

Determination of friction factor (Zhimin and Fah, 1997)

The analysis of microchannel heat sinks may cover wide flow and heat transfer regimes. The flow regimes in the channel may be laminar developing flow, fully developed laminar flow or turbulent flow. Further, the microchannel may be operated under three flow constraints such as constant coolant volume flow rate, constant pressure drop along the channel and the constant pumping power. There is a need to compute the friction factor (f) and the heat transfer coefficient (h_c) under the above mentioned flow regimes and flow constraints in order to analyse the microchannel heat sinks.

For a constant pressure drop flow constraint,

$$\Delta P = 4f\frac{L}{D}\left(\frac{\rho V^2}{2}\right) \tag{19}$$

For a fixed ΔP, the velocity, V was iterated from equations (19), (20) and (21-23) since the friction factor is a function of Reynolds number in the developing flow regime. This was achieved using Newton iteration method. The results presented in this paper were generated by using self developed programmes.

$$\mathrm{Re} = \rho_f VD/\mu_f \tag{20}$$

For laminar developing flow, the friction factor, f may be found from the following polynomial fit:

$$f\mathrm{Re} = A_0 + A_1 \ln x^+ + A_2 \ln^2 x^+ + A_3 \ln^3 x^+ + A_4 \ln^4 x^+ \tag{21}$$

where x^+ is the entrance length given by $x^+ = (L/D)/\mathrm{Re}$. The coefficients, A_i are shown in Table II for channels of different aspect ratio (α).

For fully developed laminar flow($x^+>1$), the friction factor, f can be obtained through (Philips, 1990):

$$\begin{aligned} f\mathrm{Re} = 24(1 &- 1.3553\alpha^{-1} + 1.9467\alpha^{-2} - 1.7012\alpha^{-3} \\ &+ 0.9564\alpha^{-4} - 0.2537\alpha^{-5}) \end{aligned} \tag{22}$$

For rectangular channel turbulent flow, the friction factor, f is found from the circular channel formulation as:

$$f = A(\mathrm{Re}_{eq})^B \tag{23}$$

	α = 1.0	α = 2.0	α = 5.0	$\alpha \geq 10$ and $\alpha \leq 0.1$
A_0	14.206	15.51	19.099	23.920
A_1	0.1725	0.2265	–0.445	–1.743
A_2	0.8207	0.8124	0.0409	–0.7068
A_3	0.1366	0.1539	0.00786	–0.0562
A_4	0.456	0.0479	0.0393	0.0401

Table II. Coefficients for developing flow friction factor where:

$A = 0.0929 + \frac{1.01612}{x/D]}$

$B = -0.26800 - \frac{0.31930}{x/D}$

$\mathrm{Re}_{eq} = \frac{\rho_f V D_{eq}}{\mu_f}$

$D_{eq} = [\frac{2}{3} + \frac{11}{24}\frac{1}{\alpha}(2 - \frac{1}{\alpha})]D$

Table III gives the critical Reynolds number for determining the laminar or turbulent flow regimes as suggested by Philips (1990).

For a constant coolant volume flow rate flow constraint,

$$G = nH_c W_c V \tag{24}$$

The velocity, V is known and thus the pressure drop is determined from equation (19) once the friction factor f is evaluated using equations (21-23) depending on the type of flow. The pumping power is determined as:

$$POW = \Delta PG \tag{25}$$

For a constant pumping power flow constraint, the velocity (V) is iterated from equations (20), (21-23), (24), (19) and (25).

Determination of the heat transfer coefficient (Zhimin and Fah, 1997)

For laminar thermally developing and hydrodynamically fully developed flow, the Nusselt number in a rectangular channel with four walls subjected to a constant heat flux is obtained by the polynomial curve fits written as follows:

$$Nu = B_0 + B_1 \ln x^+ + B_2 \ln^2 x^+ + B_3 \ln^3 x^+ + B_4 \ln^4 x^+ \tag{26}$$

where x^+ is the entrance length given by $x^+ = (L/D)/(\mathrm{RePr}_f)$, and Pr_f is the Prandtl number of the coolant. The coefficients B_i are shown in Table IV.

To simulate the real case, in which three walls are heated, Philips (1990) suggests that the Nusselt number be calculated using:

$$Nu_{x,3}(x^+,\alpha) \approx Nu_{x,4}(x^+,\alpha)\frac{Nu_{\infty,3}(x^+ = \infty,\alpha)}{Nu_{\infty,4}(x^+ = \infty,\alpha)} \tag{27}$$

where, $x^+ = \infty$ denotes fully-developed heat transfer.

The Nusselt number where three sides are heated for various aspect ratios have been calculated and are listed in Table V (Shah and London, 1978).

Aspect ratio	Critical Reynolds number
$\alpha = 1.0$	2,200
$\alpha \leq 0.2$ or $\alpha \geq 5.0$	2,500
Other α	Linear interpolation

Table III. Critical Reynolds number

For laminar thermally fully developed flow which occurs when the entrance length, x^+ is larger than 0.2, the Nusselt number is obtained by using (Shah and London, 1978):

$$\begin{aligned} Nu &= 8.235(1 - 2.0421\alpha^{-1} + 3.0853\alpha^{-2} - 2.4765\alpha^{-3} \\ &+ 1.0578\alpha^{-4} - 0.1861\alpha^{-5}). \end{aligned} \tag{28}$$

For the turbulent flow condition, the Nusselt number can be obtained by (Gnielinski, 1976):

$$Nu = 0.012[1.0 + (D/L)^{2/3}](Re^{0.87} - 280)\,\mathrm{Pr}^{0.4} \tag{29}$$

where Pr should satisfy $1.5 < \mathrm{Pr} < 500$.

Total thermal resistance

The problem discussed in this section concerns a forced convective flow through a microchannel whose top surface is insulated and the bottom surface is either uniformly or non-uniformly heated. A coolant passes through the microchannel and takes heat away from a heat dissipating component attached to the base of the channel. First, the case of a uniform base heat flux distribution is considered to determine the total thermal resistance using the present approach. Then the non-uniform base heat flux case is investigated.

Uniform heat flux distribution

Recent analytical results of microchannel heat sinks with uniform base heat flux distribution by Kim and Kim (1999) have been taken as the source for comparing the results obtained by the present analysis. It may be mentioned

	$\alpha \leq 0.1$	$\alpha = 1.0$	$\alpha = 2.0$	$\alpha = 3.0$	$\alpha = 4.0$	$\alpha \geq 10$
B_0	5.3570	3.5887	4.1289	4.8053	5.4214	8.30
B_1	–0.0812	0.0441	0.2150	0.1202	0.19406	–0.2593
B_2	–0.1337	–0.0417	0.02514	–0.0505	–0.0289	–0.2952
B_3	–0.0471	–0.0326	–0.0248	–0.0353	–0.0327	–0.0588

Table IV. Coefficients for developing laminar Nusselt number

$1/\alpha$	$Nu_{\infty,3}$	1α	$Nu_{\infty,3}$
0	8.235	1.43	3.195
0.1	6.939	2.0	3.146
0.2	6.072	2.5	3.169
0.3	5.393	3.33	3.306
0.4	4.885	5.0	3.636
0.5	4.505	10	4.252
0.7	3.991	∞	5.385
1.0	3.556		

Table V. Nusselt number of three-sides heated channel that Kim and Kim (1999) compared their results with those of Tuckerman and Pease (1981) and Knight *et al.* (1991). The present paper discusses the results of all the above-cited references.

Table VI gives the results in terms of the total thermal resistance of Tuckerman and Pease (1981), Kim and Kim (1999) along with the present results. Their results correspond to the optimum geometry of the microchannel heat sink. In the present calculation, the geometric dimensions given by Kim and Kim (1999) have been taken. Further, the maximum pressure difference across the microchannel heat sink and maximum pumping power given in Table VI have been used to evaluate the volume flow rate (G) which then gives the velocity V for a known number of channels (n) from equation (24). The friction factor, f was calculated using equations (21-23) depending upon the type of flow. After this the temperature distributions at the different nodes in the elements were evaluated using the finite element method described earlier. From this the total thermal resistance of the microchannel heat sink was evaluated as:

$$R_{Total} = (T_{sur,max} - T_{f,in})/(Q.L.W)$$

The same geometric dimensions and the resulting values of Re, h and Nu, as described above were also used to calculate R_{total} using the analytical

Table VI.
Comparison of present results with others

	Tuckerman and Pease (1981)	Kim and Kim (1999)	Analytical expression(2) given by Zhimin and Fah (1997)	Present method
W(cm)	1	1	1	1
L(cm)	1	1	1	1
w_w(μm)	57	59	59	59
w_c(μm)	57	59	59	59
H_c	365	378	378	378
n	88	85	85	85
POW(W)	2.27	2.27	N/A	2.27
Pressure drop(kPa)	206.8	206.8	206.8	206.8
G(cm^3/s)	11	11.3	11.3	11.3
ρ_f (kg/m^3)	N/A	N/A	997.1	997.1
Pr	N/A	N/A	6.21	6.21
μ_f(N s/m^2)	N/A	N/A	0.000891	0.000891
C_{pf}(J/kg °C)	N/A	N/A	4,179	4,179
Aspect ratio, α	6.4	6.4	6.4	6.4
V(m/s)	6.009	5.961	5.979	5.961
Hydraulic Dia(m)	0.000099	0.000102	0.000102	0.000102
Re	730	733	680.883	682.92
Nu	6	6	8.45	8.483
Fin efficiency	0.76	N/A	0.6328	N/A
R_{Total} (°C/W)	0.086	0.075	0.07	0.075

expression given by Zhimin and Fah (1997) and this calculation is also reported in the table.

Similarly Table VII shows a comparison of the results of present study with those of Knight *et al.* (1991) and Kim and Kim (1999) in terms of the total thermal resistance. Here again the geometric dimensions given by Kim and Kim (1999) were used in the present calculation. The calculated result using the expression given by Zhimin and Fah (1997) is also shown in this table.

It may be mentioned here that Tuckerman and Pease (1981) simplified the optimization problem by using various constraints such as $\varepsilon = 0.5$, a fixed Nusselt number of six, the hydraulic diameter was equal to $2w_c$ (high aspect ratio assumption) and the fin efficiency was 0.76. Further Tuckerman and Pease (1981) and Knight *et al.* (1991) used the friction factor of the channel for the pressure drop and fin theory, assuming one-dimensional conduction over the fin height for heat transfer. However, Knight *et al.* (1991) eliminated most constraints imposed by Tuckerman and Pease (1981), such as $\varepsilon = 0.5$, the fixed Nusselt number of six, the hydraulic diameter equal to $2w_c$ (high aspect ratio assumption).

Kim and Kim (1999) identified the aspect ratio and the effective thermal conductivity ratio as variables of engineering importance from their analytical

	Knight *et al.* (1991)	Kim and Kim (1999)	Analytical expression(2) given by Zhimin and Fah (1997)	Present method
W(cm)	1	1	1	1
L(cm)	1	1	1	1
w_w(μm)	60	61	60	60
$w_c\mu$m)	61	61	61	61
H_c(μm)	357	360	357	357
n	83	82	83	83
POW(W)	2.56	2.56	2.56	2.56
Pressure drop(kPa)	206.8	206.8	206.8	206.8
G(cm^3/s)	12.4	12.6	12.4	12.4
ρ_f(kg/m^3)	N/A	N/A	997.1	997.1
Pr	N/A	N/A	6.21	6.21
μ_f(N s/m^2)	N/A	N/A	0.000891	0.000891
C_{pf}(J/kg °C)	N/A	N/A	4,179	4,179
Aspect ratio, α	5.8	5.9	5.85	5.85
V(m/s)	6.86	7	6.89	6.86
Hydraulic Dia(m)	N/A	N/A	0.000104	0.000104
Re	834	826	803.4	800
Nu	Unrestricted	Unrestricted	8.63	8.62
Fin efficiency	0.76	N/A	0.66	N/A
R_{Total} (°C/W)	0.077	0.07	0.074	0.07

Table VII. Comparison of present results with others solutions based on the modified Darcy model for fluid flow and the two-equation model for heat transfer. They modeled the microchannel heat sink as a fluid-saturated porous medium. Their result for total thermal resistance was obtained from the expression derived from the analytical solutions and the geometry of the microchannel heat sink for which the thermal resistance of the heat sink is a minimum.

As shown in Tables VI and VII, the values of total thermal resistance calculated from the present finite element analysis are in good agreement with those obtained by Kim and Kim (1999) which are based on the porous medium model. The calculated results from the analytical expression of Zhimin and Fah (1997) are slightly lower in Table VI but higher in Table VII. The result of Tuckerman and Pease is higher mainly because of their assumptions as mentioned earlier. The above reason for this difference was also given by Kim and Kim (1999). As regards the comparison of result with that of Knight *et al.* (1991), the difference is 10 per cent, the same as that reported by Kim and Kim (1999). Thus it may be concluded that the present methodology is able to predict the total thermal resistance of any microchannel heat sink satisfactorily.

It may also be added that the present analysis is based on the system's approach rather than the continuum approach. Thus, the computing demand is much less compared to CFD/conjugate analysis in view of the smaller number of nodes (but with equivalent accuracy). Finally the present method can be used for thermal optimization as well.

Non-uniform heat flux distribution

In this section an attempt is made to demonstrate the capability of the present method to take into account the non-uniform heat flux distribution on the bottom surface of the microchannel heat sink. For this nine elements are considered and the heat load distribution for the different elements is shown in Table VIII. The table also shows the heat load distribution for the case of uniform heat flux for the purpose of comparison. The total heat flux ($9q = 10^6 \mathrm{W/m^2}$) imposed on the bottom surface of the microchannel heat sink for both the uniform and non-uniform heat flux distribution cases are maintained equal as can be seen from the table.

The channel dimensions, the fluid properties and the microchannel heat sink materials are the same as those mentioned in the validation section of this paper. A constant pressure drop of 5×10^4Pa was considered to take place over

Table VIII. Distribution of heat flux for different elements

Element number	1	2	3	4	5	6	7	8	9
Heat flux, W/m^2 non-uniform	$2q$	$1.75q$	$1.5q$	$1.25q$	q	$0.75q$	$0.5q$	$0.25q$	0
Heat flux, W/m^2 uniform	q	q	q	q	q	q	q	q	q

the channel length of 5,000μm. Under this flow constraint, the value of the overall heat transfer coefficient was determined following the method described previously.

Figure 4 shows the predicted temperature distribution of the bottom surface of the heat sink along the length of the microchannel for both the uniform and non-uniform heat flux distributions for different elements. It is seen that for uniform heat loading, the bottom surface temperature varies from 15.81°C to 20.16°C for the fluid inlet temperature of 10°C. When the heat flux is distributed non-uniformly as mentioned in Table VII, the variation in the above mentioned temperature is from 21.88°C to 15.07°C for the same fluid inlet temperature. The bottom surface temperature at the inlet to the microchannel heat sink is highest because the heat load given to the first element of the channel is twice that of the uniform heat flux distribution case. The above temperature distribution in the second half of the channel is lower than that of the uniform heat load distribution because the heat loads distributed to elements beyond five are lower as compared with the uniform heat load distribution.

Figure 4 also shows the coolant temperature variation along the length of the microchannel heat sink for both cases of heat loading. A rise in the coolant temperature along the microchannel length is clearly seen for both the cases of heat loading. At the end of the channel, the coolant temperature is the same (14.68°C) for both the cases. This result is as expected because the sum of the heat fluxes for all the elements for both the cases considered is the same. The total thermal resistance of the microchannel heat sink is found to be 0.102 °C/W-cm^2 (0.102×10^{-4} °C/W-m^2) for uniform heat flux distribution and 0.119 °C/W-cm^2 (0.119×10^{-4} °C/W-m^2) for non-uniform heat flux distribution respectively. It can be observed from this that the non-uniform heat flux distribution increases the thermal resistance by about 17 per cent compared with the uniform heat flux distribution.

Similar calculations may be carried out for any other heat flux distribution and the total thermal resistance of the microchannel heat sink can be found for that distribution. It may be mentioned here that even different heat loads at different node points may be accounted for in the present methodology. Thus

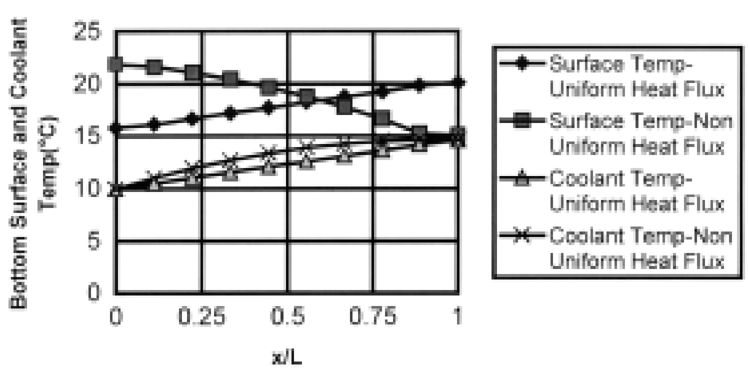

Figure 4.
Variation of bottom surface and coolant temperatures for uniform and non-uniform heat flux distributions

the present methodology has the advantage of taking into consideration any form of non-uniform heat flux distribution on the bottom surface of the microchannel heat sink as compared to other available methods mentioned earlier.

Conclusions

A Galerkin finite element formulation has been developed to study the performance of a microchannel heat exchanger that is used in electronic package cooling. The present method has been validated successfully against the available results of uniform heat flux distribution for the same channel geometry and fluid flow conditions. The proposed methodology has an additional advantage of considering the non-uniform heat flux distribution as well. This aspect is illustrated by an arbitrary example which results in an increase of the total thermal resistance by 17 per cent for the non-uniform heat flux distribution considered. The present method is also useful to study the effect of parameters such as the channel height, channel width, and the fin thicknesses on the performance of the microchannel heat exchanger. Thus the present analysis may also be considered to be a useful tool for optimum thermal design of microchannel heat sinks.

The method used is an alternative to massive CFD calculations. In view of the smaller number of nodes in the present system's approach, the computing time is much less compared with that for the CFD/conjugate analysis.

References

Copeland, D., Behnia, M. and Nakayama, W. (1996), "Manifold microchannel heat sinks, conjugate and extended analysis", *9th Symp. Trans. Phen. Thermal-Fluid Eng.*, Singapore, pp. 498-503.

Daane, D.A. and Franzon, P.D. (Ed.) (1993), *Multichip Module Technologies and Alternatives: The Basics*, Van Nostrand Reinhold, New York, NY, pp. 569-613.

Gnielinski, V. (1976), "New equations for heat and mass transfer in turbulent pipe and channel flow", *International Chemical Engineering*, Vol. 16, pp. 359-68.

Goldhaber-Gordon, D., Montemerlo, M.S., Love, J. C., Opiteck, G.J. and Ellenbogen, J.C. (1997), "Overview of nanometer devices", *Proc. IEEE*, Vol. 85 No. 4, pp. 521-40.

Harpole, G.M. and Eninger, J.E. (1991), "Microchannel heat exchanger optimization", *Seventh IEEE Semi-Therm. Symposium*, pp. 59-63.

Kim, S.J. and Kim, D. (1999), "Forced convection in microstructures for electronic equipment cooling", *ASME J. Heat Transfer*, Vol. 121, pp. 639-45.

Knight, R.W., Goodling, J.S. and Hall, D.J. (1991), "Optimal thermal design of forced convection heat sinks-Analytical", *ASME J. Electronic Packaging*, Vol. 113, pp. 313-21.

Knight, R.W., Hall, D.J., Goodling, J.S. and Jaeger, R.C. (1992), "Heat sink optimization with application to microchannel", *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*, Vol. 15 No. 5, pp. 832-42.

Koh, J.C.Y. and Colony, R. (1986), "Heat transfer of microstructures for integrated circuits", *Int. Comm. Heat Mass Transfer*, Vol. 13, pp. 89-98.

Leng, C.S. (1997), "Electronic cooling using microchannels", Final Year Project, BSc(Hon), School of Mechanical Engineering, USM(KCP), Malaysia.

Lewis, R.W., Morgan, K., Thoma, H.R. and Seetharamu, K.N. (1996), *The Finite Element Method in Heat Transfer Analysis*, John Wiley and Sons, New York, NY.

Mizunuma, H., Behnia, M. and Nakayama, W. (1997), "Heat transfer from micro-finned surfaces to flow of fluorinet coolant in reduced-size channel", *IEEE Transactions On Components, Packaging, and Manufacturing Technology – Part-A*, Vol. 20 No. 2, pp. 138-45.

Philips, R.J. (1990), "Microchannel heat sinks", in Bar-Cohen, A. and Kraus, A.D. (Eds), *Advances in Thermal Modeling of Electronic Components and Systems*, ASME, pp. 109-84.

Ranganayakulu, C. and Seetharamu, K.N. (1999a), "The combined effects of longitudinal heat conduction, flow non-uniformity and temperature non-uniformity in cross flow plate-fin heat exchangers", *Int. Comm. In Heat and Mass Transfer*, Vol. 26, pp. 669-78.

Ranganayakulu, C. and Seetharamu, K.N. (1999b), "The combined effects of wall longitudinal conduction, inlet fluid non-uniformity and temperature non-uniformity in compact tube-fin heat exchangers – a finite element method", *Int. Journal of Heat and Mass Transfer*, Vol. 42 No. 2, pp. 263-73.

Salman, V.K. (1989), "Convective heat transfer in microchannel", *Journal of Electronic Materials*, Vol. 18, pp. 611-18.

Segerlind, L.J. (1984), *Applied Finite Element Methods*, John Wiley & Sons, New York, NY.

Shah, R.K. and London, A.L. (1978), "Laminar flow forced convection in ducts", *Advances in Heat Transfer*, Academic, New York, NY, pp. 196-223.

Tien, C.L. and Kuo, S.M. (1987), "Analysis of forced convection in microstructures for electronic system cooling", *Proc Int. Symp. Cooling Technology for Electronic Equipment*, Honolulu, HI, pp. 217-26.

Tuckerman, D.B. and Pease, R.F.W. (1981), "High performance heat sink for VLSI", *IEEE Electron Device Letters*, Vol. EDL-2 No. 4, pp. 126-9.

Vafai, K. and Tien, C.L. (1981), "Boundary and inertia effects on flow and heat transfer in porous media", *Int. J. Heat Mass Transfer*, Vol. 24, pp. 195-203.

Wang, B.X. and Peng, X.F. (1994), "Experimental investigation on liquid forced-convection heat transfer through microchannel", *Int. J. Heat Mass Transfer*, Vol. 37 Suppl. 1, pp. 73-82.

Wiesberg, A., Bau, H.H. and Zamel, J. (1992), "Analysis of microchannels for integrated cooling", *Int. J. Heat Mass Transfer*, Vol. 35 No. 10, pp. 2465-74.

Yeung, T.S., Yeun, M.M.F. and Zohar, Y. (1995), "Heat transfer in microcooling system", *6th Asian Conf. Fluid Mech.*, Singapore, pp. 785-8.

Zhimin, W. and Fah, C.K. (1997), "The optimum thermal design of microchannel heat sinks", *IEEE/CPMT Electronic Packaging Technology Conference*, pp. 123-9.